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Bryan Jack and Mancur Olson[1]

One of the more intriguing logical discoveries in economics in recent years is the result, due to Peter G. Warr (1983) and foreshadowed and developed by others,[2] of "neutrality": the finding that "when a single public good is provided at positive levels by private individuals, its provision is unaffected by the distribution of income." A redistribution of income among the set of individuals voluntarily contributing to the provision of a public good leaves each person with exactly the same amount of both the public and the private good as before the redistribution because the beneficiaries of the redistribution increase their expenditures by exactly the same amount by which the losers from the

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[2] At the time that Warr's paper on neutrality was published, a number of other economists had independently been heading toward this result. Shibata (1971) approached the result in his study of Pareto-efficient provision of public goods. Becker, in his "rotten kid" theorem, demonstrated a type of neutrality in the presence of a redistributive parent. And, Cornes and Sandler had by 1983 discovered a form of the neutrality result in working papers preceding their 1985 article on the analytics of public good provision.

redistribution reduce theirs. Bergstrom, Blume and Varian (1986) describe Warr's result as "striking" and offer generalizations of it. B. Douglas Bernheim (1986) calls the result "startling" and argues that, even as generalized by Bergstrom, Blume and Varian, it is "only the tip of the iceberg": maintaining Warr's assumptions, he demonstrates that any public policy of contributions or transfers to a privately provided public good, whether financed by lump-sum or discretionary taxes, has no effect on the amount provided or on resource allocation generally.

When rigorous deductions lead to conclusions as puzzling as these, it often turns out that there is more to the story. We shall see in this paper that this is, indeed, the case here. When the recent discoveries are properly understood and combined with some older theory, this leads to more general results that are both intuitively and practically satisfying. This paper will initially compare the testable implications of the neutrality theory with observed reality and then develop a more general conception.

Empirical testing of the theory growing out of Warr's work is trivially easy, since the theory predicts a pattern of behavior that almost certainly could not arise from other causes. This amazing pattern is also most unlikely to pass unnoticed. Yet our survey of the literature did not reveal even one example of the change in the pattern of contributions to a public good that the theory tells us would result from a redistribution of income among the contributors to a public good. No one we have talked to has been able to think of any observation or empirical test that is likely to support the theory. This does not, however, mean that the neutrality literature has no value -- when it is properly combined with other work, we obtain a more general theory that is in full accord with the empirical evidence.

II

The neutrality theory, especially as extended by Bernheim, also makes startling and testable predictions about whether the governmental provision of public goods will increase the total amount of public goods provided or instead be wholly offset by a reduction in private provision. Warr (1982) appeared to demonstrate that, though the free rider problem implies that private charity leads to a less than Pareto-optimal level of provision, governmental redistribution will simply reduce private giving dollar-for-dollar and will change the level of provision only if it exceeds the level where private charity ceases. Roberts (1984) deduced similar conclusions.

Bergstrom, Blume and Varian's aforementioned analysis, which makes a major advance over the rest of the neutrality literature by emphasizing that the neutrality results hold only if the set of contributors remains unchanged, points out that the Warr and Roberts results apply only if all the taxes are collected from people who are contributors to the public good. Bergstrom, Blume and Varian contend that, in fact, those who privately contribute to the provision of the public good are normally only a subset of the taxpayers, so that the taxes paid by contributing consumers are less than the total tax collections, and governmental provision can then increase the total level of provision of a public good. Bernheim, on the other hand, demonstrates that, if there is sufficient overlap among contributors to different public goods, then government provision of a public good adds nothing to the supply even if some taxpayers had not been contributing any of the public good.

Thus the neutrality theory in some formulations implies that the increase since World War II in governmental expenditures on public goods, whether for welfare state or military

purposes, has had no effect on the amount of welfare or defense that has been provided! Bernheim is much too prudent to draw this bizarre conclusion. He argues instead that the real implication of the theory is that individuals who contribute to the provision of a public good do not do so simply because they care about the aggregate level of expenditures on that public good -- they must instead contribute to the provision of public goods at least partly for other reasons.

Whatever other reasons Bernheim has in mind must be quite different from the motives that economists normally suppose govern resource allocation. Possibly citizens contribute to the provision of a public good not only to increase their consumption of it, but also because they gain moral satisfaction from making voluntary sacrifices to provide a public good that others also enjoy. They then contribute even more than they would when they are motivated only by their desire for more consumption of the public good. This would make the purely private provision of public goods even larger than the Bernheim-Warr type of analysis implies and the contrast between the theoretical predictions and the observed facts even starker.

III

A number of writers, such as Bergstrom-Blume-Varian and Cornes and Sandler (1986), trace the literature on the private provision of public goods back to Olson's *Logic of Collective Action* (1965). But, at least in most cases, previous writers do not relate the

results growing out of Warr's discovery to the theory in that book and in Olson-Zeckhauser (1966), even though this earlier theory also makes predictions about the sharing of the burden of a voluntarily provided public good. Most notably, the older theory predicts that the voluntary provision of a public good leads to what Olson (1965) called the "exploitation of the great by the small" -- to a sharing of the burden of providing a public good that is dramatically unfavorable to the party with the largest absolute demand it. Thus, for example, in a defense alliance such as NATO or in a cartel such as OPEC (where a higher price is a collective good to oil exporters) the country with the largest absolute demand for the collective good at issue is predicted to bear a disproportionate share of the burden of obtaining it. There have been many empirical studies of this implication of disproportionality and almost all of them provide at least some support for it (for citations to this literature, see Oneal and Elrod [1989] and Sandler [1991]).

The Warr-neutrality theory and the theory implying disproportional burden sharing seem to contradict each other, but in fact they do not. Both theories use the same behavioral assumptions and they can readily be integrated into a more general theory. When this is done, it is immediately evident why it is so difficult to find real-world examples of the neutrality result. The more general conception of the matter also reveals a remarkable "natural egalitarianism" in the voluntary provision of public goods -- that is, a powerful tendency for this type of provision to reduce inequalities in real income or welfare.

To derive the more general theory and even to obtain a full understanding of the Warr literature, we must consider the entire range of possible distributions of resources in a group that can enjoy the voluntary provision of a collective good. Warr considers only

redistributions that leave the set of contributors unchanged, and Bergstrom, Blume and Varian show most impressively how crucial this restriction is. But none of the articles on the neutrality result explores what would happen across all possible allocations of the wealth in the group that consumes a public good. In a group of two individuals, for example, one of the two might have (or in a redistribution be given) all of the wealth, or the other might possess (or obtain) all of it, or there could be any one of the infinite set of divisions of endowments between these two polar extremes.

We must also define and compare the "unilateral" and the "full income" quantities of a public good demanded by each of the parties in a group and then compare each of these quantities with the corresponding quantities demanded by each of the others. The unilateral demand for a public good is the amount that a party would purchase in isolation or when no amount of this good is provided by any other party, and it has been analyzed at least since Olson's (1965) account of the "privileged" group. The full-income quantity demanded by a party is the level of provision it demands when its opportunity set is enlarged by the free provision of the public good by one or more of the other parties, and it has also often been analyzed in the literature.

IV

In considering the quantities of a public good that are demanded by each of the parties it is convenient to start with only two parties. The implications of our analysis for larger groups will be obvious. We also assume, as do Bergstrom, Blume and Varian (1986), that the public good is a normal good, i.e. has an income elasticity of demand greater than zero.

With two parties, i and j, each of which could in the appropriate circumstances demand either a unilateral or a full-income quantity, all the logical possibilities can be dealt with quickly:

(1) The quantity of a public good demanded unilaterally by i exceeds the full income demand for this public good of j, $U_i > F_j$. In this case, even though j's income has been increased by the free provision of the public good by i and the public good is a normal good, j does not wish to purchase any of the public good. Party j might, if this were possible, like to sell some of the public good and use the income received to purchase more of the private good, but with a public good this is impossible by definition. Thus we have in this case a stable Cournot-Nash equilibrium with disproportional burden sharing -- i is stuck with the full bill.

(2) The exact opposite of the above case: $U_j > F_i$. This is also a perfectly possible Cournot-Nash equilibrium and there is again disproportionality in burden sharing, with j instead of i being stuck with the entire burden.

(3) Crucially, there is no possible two-party Cournot-Nash equilibrium with either party demanding a greater full-income quantity than the other. By the definition of a public good, both parties get the same amount, so if one party were to demand a larger full-income quantity than the other, the other party would then have this larger amount of the public good and then provide none at all, in which case the relevant provision of the first party would be the quantity it would provide unilaterally.

(4) Obviously, two (or more) parties cannot, by the definition of unilateral provision, simultaneously provide unequal quantities.

(5) There is only one remaining logical possibility: the two parties might have exactly the same full-income standards for the public good, $F_i = F_j$.[3]

As we shall soon see, there is normally only a quite restricted set of circumstances in which this condition is satisfied, and that is the reason the neutrality result has apparently not been observed.

V

Our analysis builds upon an analytical and expository technique developed by Bryan Jack (August, 1990), which can simultaneously explain the neutrality result and the logic of disproportionality in burden sharing with a simple set of diagrams.[4] Jack supposes there are two actors, i and j, who value a public good, and that the production functions and factor prices of i and j are the same, so that a unit of the public good has the same cost, at every level of output of the public good, to each of them.

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[3] If there are three (or more) parties, it is possible for two (or more) of them to have the largest full-income demand, $F_i = F_j$, for the public good, so that each provides some positive amount of it, while one (or more) other party has a lesser full-income demand $F_k < F_i = F_j$, and thus provides nothing.

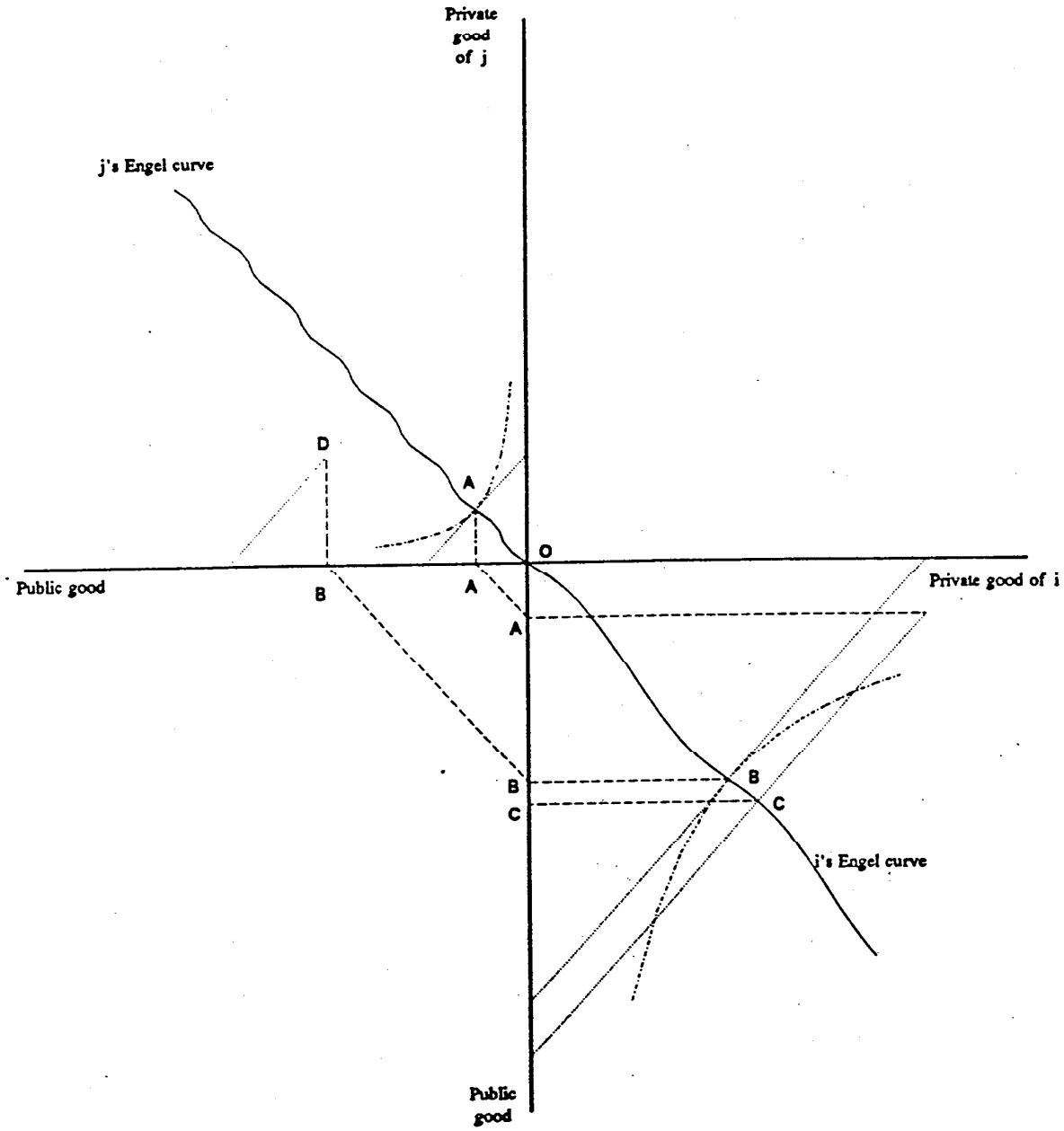
[4] Just as several researchers converged upon the neutrality result in the 1970s and early 1980s, there are several diagrams in the literature, each expressing neutrality in a different context. Shibata (1971, figure 6) uses an asymmetrical diagram, with reaction curves, showing Cournot-Nash equilibria; it can easily be used to illustrate the neutrality result. McGuire (1974, figure 1) offers a diagram to show the impact of adding a second potential public-good contributor to a one-member "group, and the neutrality result can be seen in that diagram as well. Cornes and Sandler (1985) presented a simple diagram, in public-good space, that demonstrates neutrality by means of a sliding X-Y intercept. The important thing is that each of these diagrams has a different intended application from those presented here. The diagrams developed in this paper are intended to explore neutrality in the spectrum of all possible distributions of a given community endowment.

Figure 1 depicts an allocation of initial wealth such that j will provide none of the public good and will free ride on i's level of provision. If j were to provide his isolation output of the collective good, OA, this amount would be available to i, and i's budget constraint would be shifted out so that he could buy whatever he bought before plus OA of the public good. With this budget constraint he would purchase OC of the public good, or much more than OA. With this level of provision, j would provide none, so this is not an equilibrium.

If i were to purchase his isolation output of the public good, OB, then this amount would be available to j, who would then choose to spend all of his disposable income on private goods and be in equilibrium at point D, wishing that it would be possible for him to consume a larger proportion of his income in the form of private goods. This is a Nash-Cournot equilibrium for both parties, so this outcome, with free riding by j, would be the only Nash-Cournot equilibrium. Obviously, Figure 1 can be modified to treat the case of a wealthy j and a more modestly endowed i, with the same utility functions and Engel curves as before, to show an outcome where j provides all the public good and i free rides.

If, as we continue to assume, the collective good is never an inferior good, then the full-income demand of a party must exceed its unilateral demand, simply because its full income exceeds its isolation or unilateral-provision income. Thus if one party's unilateral demand for a collective good exceeded the other party's full-income demand, the first party's full-income demand would also exceed the other party's full-income demand. Therefore, we need no further analysis of cases where the unilateral demand for the collective good of one

Figure 1


Private good of j
j's Engel curve
D
A
O
Public good
B
A
Private good of i
A
B
C
B
C
j's Engel curve
Public good

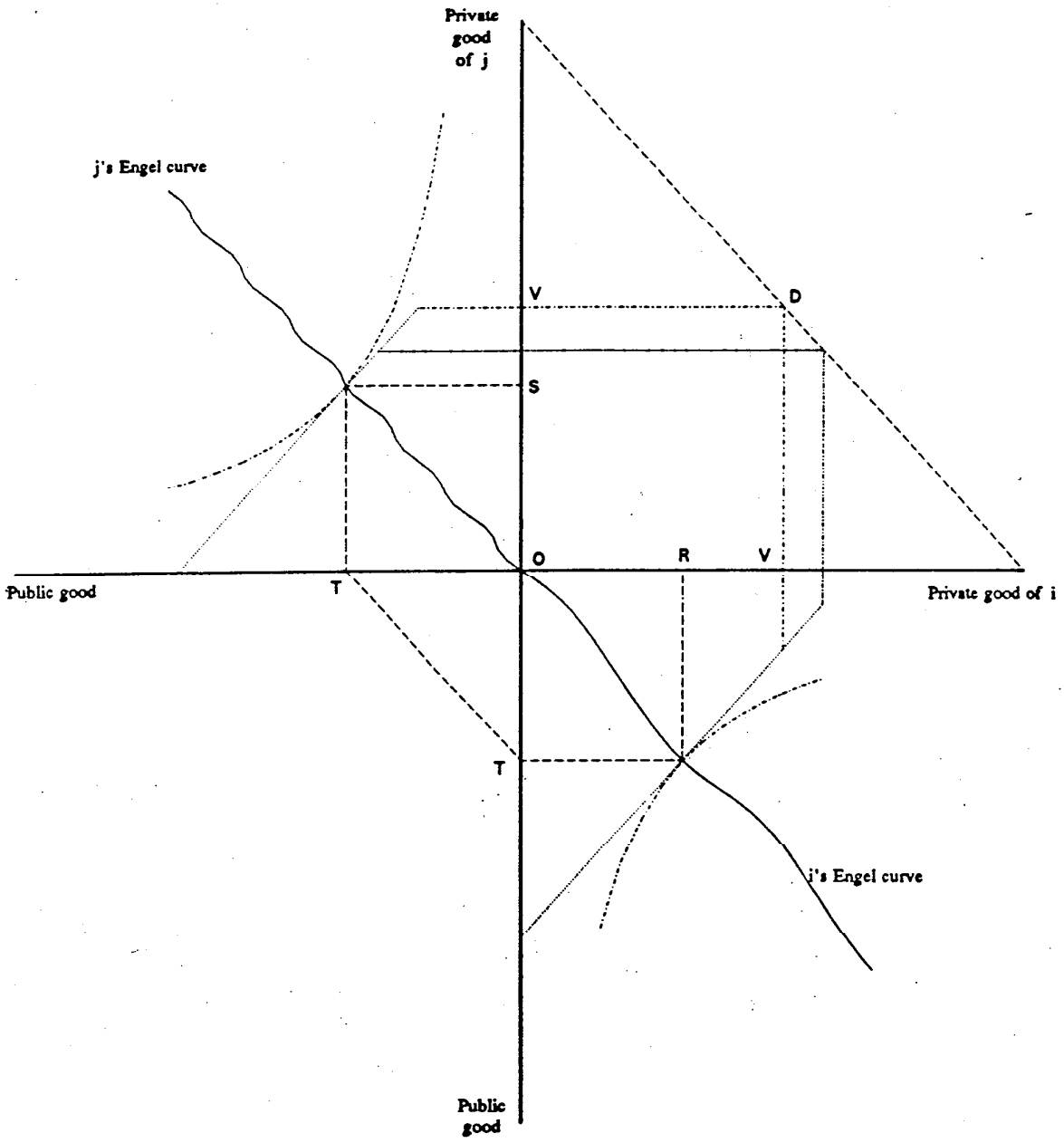
party exceeds the full-income demand of the other: these cases involve unsymmetrical burden sharing but do not lead to a neutrality result.

VI

With this exposition, we can explain the neutrality results in the literature growing out of Warr's work in an astonishingly simple and intuitive way. Let us suppose that there are two parties and a fixed endowment of private goods, which may be divided in any manner between the two parties. Let us shift the distribution of the initial endowment in Figure 1 in the direction of the free riding party, and shift it to such an extent that the unilateral provision of the provider party no longer exceeds the full-income provision of the other. Indeed, let us shift it just enough to ensure that both i and j have exactly the same full-income demands for the public good. This leads to the situation we depict in Figure 2, which is the same as Figure 1 except that, in the northeast quadrant, all possible allocations of the private good of the society are given along the dashed line with a slope of -1; any point on this line (w_i,w_j) will depict the private good wealth of each of the parties. At point D in Figure 2 the distribution of initial endowments for i and j is such that it will give both parties precisely equal full-income demands; the point D may conveniently be found by arbitrarily starting with the level of public good provision that has both parties in equilibrium providing positive amounts of the public good and then finding the initial endowments of each that are consistent with this double interior-solution equilibrium.

At the level of public provision depicted, party i consumes OR and party j OS of the private good. Party j spends SV on provision of the collective good and i spends RU on it.

Figure 2


Private good of j
j's Engel curve
V
D
S
O
R
V
Public good
T
Private good of i
T
j's Engel curve
Public good

These expenditures respectively exhaust the endowment income of each and the budget constraints are drawn to reflect the fact that neither party can possibly consume less of the public good than the amount provided by the other.

The intuitive essence of the Warr result may be seen with striking simplicity by examining the impact, at this distribution of endowment income between the two parties, of a transfer of private goods from one party to the other. Suppose, as shown on the figure, a small transfer of private goods from j to i. The result is that, as j is made "poorer" and i made "richer" by this transfer of the private good, j "exploits" i's public good provision to get a "public good" transfer of exactly identical value, so that there is in fact no change in the distribution of income or welfare. Party i becomes "greater" and party j "smaller" just enough to exactly compensate, both in distribution and allocation effects, for a transfer of private good from j to i.

The most important implication of this point is evident only when it is integrated with earlier findings. As we shall wee, the voluntary provision of public goods is inherently egalitarian. The familiar result from the neutrality literature arises, in the special cases where it does in fact arise, only because the "egalitarian" or exploit-the-bigger-demander burden sharing logic exactly offsets the impact of a redistribution of the private good. In our example in Figure 2, individual j was made "poorer" and individual j "richer" by a small transfer of private goods. But the party made poorer is not even a trifle worse off, since the party made richer increases his provision of the public good by exactly the amount the loser reduced his, and both have precisely the same level of public good consumption and public

good provision as before. We shall see that this is just a snowball on the tip of an iceberg of natural egalitarianism.

VII

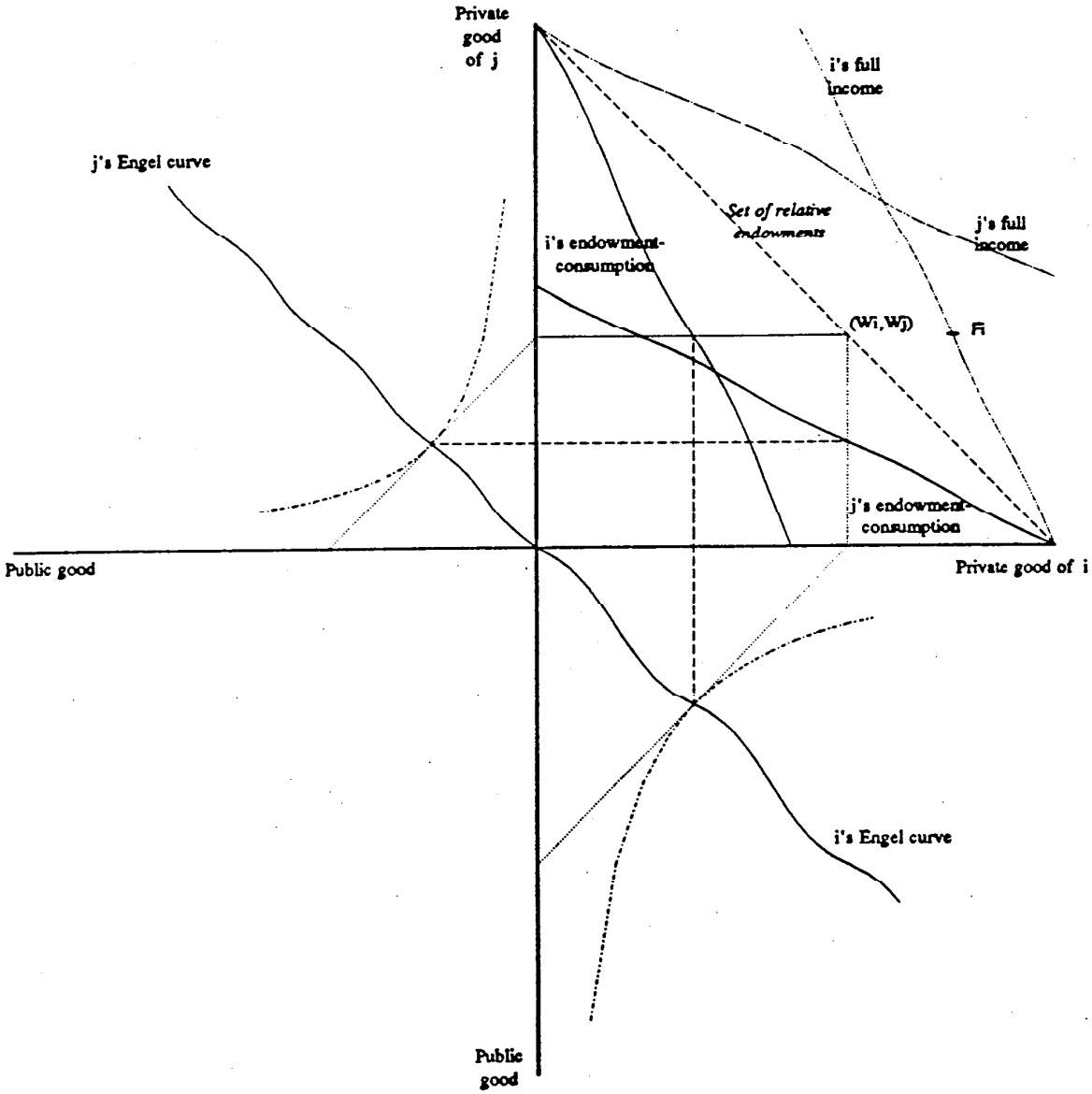
Exactly when can the result that the Warr literature has made famous occur? It is already clear that both (all) parties must demand exactly the same level of public good provision, but when will this happen? This is shown in Figure 3, drawn from Jack (1990), which in the northeast quadrant contains projections of both the Engel curves and the enlarged-opportunity or full-income demands for i and j.

The Engel or endowment-consumption curves are constructed by taking points on the locus of possible distributions of endowments and plotting each member's isolation expenditure on the private good as a function of the other member's endowment. From any given endowment point one can read across to determine i's isolation expenditure levels and read down to determine j's. At the illustrative point (w_i,w_j) shown, the isolation expenditures of each of the respective parties are depicted in the northwest and southeast quadrants.

From the endowment-consumption curves we can also calculate the resource cost of whatever amount of the collective good each party would get at each point because of the other's provision. By going outward from the 45-degree line by this amount we are able to depict the "full income" that each party would have at each distribution of the endowment. To calculate i's full-income curve, we reflect j's endowment-consumption curve in the 45-degree line. The vertical distance from j's endowment-consumption curve to the 45-degree

Figure 3

Construction of Endowment-Consumption Curves



line represents the amount of j's income that it has spent on the public good rather than the private good. Thus it indicates the portion of i's income that is due to j's provision of the public good. At the endowment point (w_i,w_j) shown, the hypothetical or isolation public good output that j is depicted as producing at that endowment point would have given i a full income greater than that at (w_i,w_j) by an amount that can be represented as a horizontal distance to the right of (w_i,w_j) equal to the vertical distance between j's endowment-consumption curve at that point, *i.e.* to point F_i. This point is on i's full-income line, which obviously starts at the point on the 45-degree line where i has all the income and j thus necessarily produces no public good. Naturally i's full-income curve goes further to the right of the 45-degree line as j's endowment income and hypothetical public good provision increase.

We shall here leave aside certain complexities that have no importance for the issue at hand, such as those that can arise from decreasing or varying marginal costs of producing the public good or preference orderings in which the public good over any range is an inferior good. Thus, in keeping with most of the literature growing out of the Warr result, we assume that all parties have the same constant marginal costs of production of the public good and that there are no discontinuities in production or utility functions. These assumptions entail that the consumption-income and full-income functions are smooth and cross only once.

Since we are considering all possible distributions of endowments for the two individuals, our assumptions also imply that there are an infinite number of possible levels of provision of the collective good. There could be one distribution of income at which i

unilaterally provided a given quantity of public good and another more or less opposite distribution of income at which j unilaterally produced exactly the same quantity of public good. But apart from this there would be, under the assumptions we have made, a different level of public good provision for each distribution of income. At each level of public good provision, whether it is provided unilaterally or with contributions from both parties, there is at least one distinct pair of quantities (one for i and one for j) of private good consumption. Since we have excluded discontinuities there is an infinite set of these pairs of private good consumption levels as well as this infinite set of levels of provision of the public good.

VIII

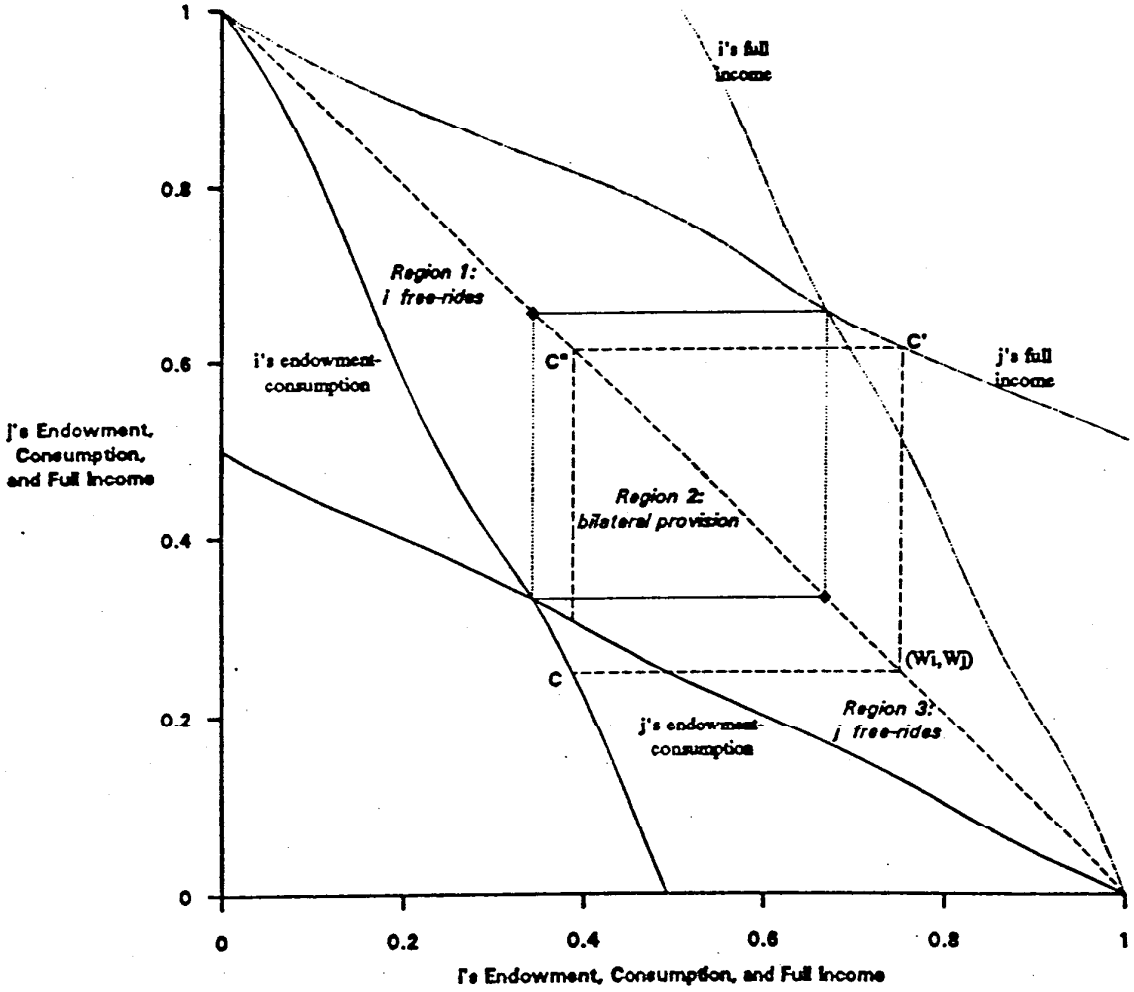
We can see how often the type of result analyzed in the neutrality literature comes up by going to Figure 4. It will be helpful at this point, especially for our later discussion of natural egalitarianism, to suppose that the two parties have identical preference orderings. This will conveniently rule out the possibility that the party with the lower income would have the larger demand for the public good and make it possible to identify the "richer" party as the one with the larger demand for the public good.[5]

We begin at the point where i has all of the income and then see what happens as we shift the distribution progressively toward j. When i has all the income his private good consumption is shown at the point where his endowment-consumption or Engel curve intersects the horizontal axis. His spending on the public good is the remainder of the social

[5] Figure 4 does not, however, depend upon an assumption that parties i and j have identical preference orderings.

Figure 4

Cournot-Nash Public-Good Provision Regions



income and is measured by the horizontal distance to the 45-degree line. This means that j gets a large amount of the public good as a free rider. Of course, j would like to trade the public good for the private good, but this is impossible by definition for a public good.

As we move to a less unequal distribution of endowments such as point C, then i's free public good provision to j is decreasing (the horizontal distance to the 45-degree line is shortening), and j obviously has more private good. But j still does not have an equilibrium amount of the private good, as we can see by considering his full income, which is his amount of private good -- the amount given by the (w_i,w_j) locus corresponding to point C -- plus the substantial gift of the public good from i, which takes j to point C' on his full-income curve. As the dotted lines show, at C' j is as well off as he would have been had he been at C" on the 45-degree line but received no gift of the public good.

How much public good would j have purchased at C"? We can see from j's endowment-consumption curve that his equilibrium private good consumption would have been at the point on his endowment-consumption curve directly below C". This is an amount that is a little greater than j is in fact consuming at allocation C. So it follows that j is consuming a larger proportion of his income in public goods than he would like to. Thus, to go back to our earlier exposition, the unilateral demand of one party for the public good exceeds the full-income demand of the other and there is no neutrality result.

Now let us shift the distribution of endowment away from i, and by just enough to bring him to the point on his endowment-consumption curve where it intersects j's corresponding curve. Here the same procedure as we used at point C will take us to a point on j's full-income curve such that, if he had endowment income just equal to this and no gift

of a public good, he would have ended up purchasing precisely the amount of private good he is now consuming. So j now has an equilibrium supply of the public good. This is the very amount that i is demanding, so we have reached a situation in which both happen to demand exactly the same level of provision of the public good.

This is just the point where the natural egalitarianism that, as we shall see, characterizes the whole range of voluntary Nash-Cournot public good provision, takes on a new form. At the point we have just considered, i has more of the endowment than j: to a casual observer he would be "richer". But the natural egalitarianism ensures that i isn't really any richer than j: at the point we have just considered i pays the whole bill for the public good. And, given our temporary assumption of identical preferences, the two have precisely the same level of real income or utility. The modest difference in endowment income is totally offset by the incentive that the "richer" party has to pick up the entire check for the public good!

Now, let us shift the distribution of endowments in favor of j just enough to make him "richer" than i by the corresponding amount -- that is, put him at the allocation of endowment such that i is right on the margin between joining in providing the public good and being a total free rider. Of course, at this point the exploitation of the great by the small goes in just the other direction: j's additional endowment is totally offset because he now has an incentive to pay the whole bill for the public good, so the two incomes are still exactly equal.

The natural egalitarianism is such that, so long as one stays inside the "square" introduced by Jack, any redistribution of endowment will have no effect on real income or welfare: the exactly equal distribution will prevail over the range.

This is, of course, also the Warr result. If we rule out the possibility that the poorer party has the greater taste for the public good, we can restate the Warr result in a way that gives a very different impression. It then comes down to the idea that, in the special circumstance when more than one party is providing the public good, the burden of the costs of the public good will always be such as to reduce any inequality of real income or welfare among the contributors. If tastes are identical, the Warr result applies only when the distribution of income or welfare is exactly equal. We shall see later that other manifestations of natural egalitarianism with voluntary public good provision are normally much more important than the one that has just been described.

IX

Let us now drop the assumption of identical tastes and return to the same assumptions that are typical of the rest of the Warr literature so that we can meaningfully ask how often the Warr result applies. Our analysis luckily makes it possible for us to explain why no one has evidently been able so far to find a real-world example of this phenomenon.

Our exploration of the implications of the whole range of possible income distributions between two people has revealed that the Warr result holds at only one level of public good provision. It also holds at only one pair of consumption levels for the private good: the endowments change within the square in Figure 4, but not either the level of

public good provision or the private good consumption of either party. There is also only one distribution of real income or welfare -- the one that is consistent with the level of public good provision and the pair of private good consumption levels given by the intersection of full income curves or (what is the same thing in this analysis) the intersection of the endowment-consumption curves.

The neutrality result is therefore normally applicable in only a restricted set of circumstances. Of course, different cost and utility functions could make the endowment-consumption and full-income curves intersect more than once, and sometimes also in ways that would give more than one intersection that is a stable equilibrium. But even when this is the case there will be at most only a few levels of public good provision, pairs of private good consumption levels, and real income or welfare levels at which the Warr result will hold.

These results are easily extended to groups with more than two members. The Warr-neutrality result can occur in a group of n members only if there is one level of public good provision and set of private good consumption levels that leaves every member of the group in Nash-Cournot equilibrium. Otherwise, the only Nash-Cournot equilibrium with provision of the public good occurs when the party with the largest absolute demand for the public good is at its unilateral optimum and all the other parties enjoy a free ride. Thus the likelihood that there will be a Warr-neutrality result that applies to all members of a group gets ever smaller as the group gets larger. (As Olson [1965] demonstrated, the group also falls farther and farther from achieving a group optimal level of provision as the number of members increases.)

X

There is, however, a limited range of endowments that is consistent with each level of public good provision and pair of private good consumption levels. The Nash-Cournot private provision of public goods is never independent of the distribution of income -- that is, it is never independent of the distribution of real or true income or welfare at which the neutrality result holds -- but it can, within a range, be independent of the distribution of endowments. Though this range is normally rather narrow, there are certain circumstances when it is wider. Even for a two-member group, modest (and, possibly, ordinary) levels of interest in consuming a public good imply very egalitarian income-distribution conditions for the neutrality result. For example, suppose that the two members have identical tastes in the form of Cobb-Douglas utility functions. If the tastes are such that each party (in isolation) would spend 90% of income on the private good and 10% of income on the public good, then the only distributions of income that produce the neutrality result are those where the poorer party has at least 47.4% of the total endowment. In a more extreme case of preference for public goods, suppose that each party has Cobb-Douglas utility functions giving equal weight to private and public goods. Thus, in isolation, any member of the group would spend 50% of his or her endowment on the private good and the other 50% on the public good. In that case, the neutrality result occurs only if the poorer party has no less than 33.3% of the total endowment.

It is also important to understand that as the population size increases, the already limited likelihood that the Nash-Cournot equilibrium exhibits the neutrality result becomes vanishingly small. Andreoni (1988) has shown, in a simulation of income distributions and

charitable contributions, that in large populations the large majority will free-ride. A simple numerical example can demonstrate how any neutrality is extinguished when group membership increases. Let us continue to suppose that every member of an n-member group has identical tastes, amounting to a Cobb-Douglas utility function that gives equal weight to the private and to the public good. This means that no income distribution in which any member has less than $1/(n+1)$ of the total income of the total income will produce the neutrality result. If we further assume that all income distributions are equally likely (*i.e.*, that the density function of income for an n-member group distribution is uniformly distributed over the (n-1)-dimensional unit simplex of relative income), then the likelihood of neutrality for a group of n members is $[1/(n+1)]^{(n-1)}$. For $n = 2$ this likelihood is 1/3, but for $n = 3$ it is 1/16, and for $n = 4$, it is 1/125.

XI

Natural egalitarianism is, if anything, more striking in the more common cases where the Warr-neutrality result does not occur than it is in those special cases. To understand this natural egalitarianism in all its simplicity, it is useful to abstract once again from differences in preferences. The natural egalitarianism is also normally important when there are differences in tastes, but it can be obscured or on occasion even offset by a tendency for the "exploitation of the zealous by the apathetic." We will analyze this tendency later and so we return now to Figure 4 and again consider the entire range of possible distributions of income. It is immediately obvious that when i has all of the income it will bear the entire cost of the public good and that this will reduce the inequality in welfare.

The more important point is that, at all distributions of the endowment except an exactly 50-50 split, the party with the larger endowment will either (in the more common case) reduce inequality by bearing all of the cost of the public good, or (in the less common cases with neutrality) bear whatever share of the cost he needs to bear to ensure that the distribution of true income becomes perfectly equal. In any society (unless it already has perfect equality in endowments) the Nash-Cournot private provision of public goods is always inequality reducing: it either reduces the degree of inequality or else generates an exact equality of welfare out of an inequality of endowments. It will also partly or (in the less common case where there is continuing neutrality) totally countervail any inequality-increasing redistributions of the endowment. The "winner" in any redistribution of sufficient size is also the total loser who gets stuck with the whole cost of the public good.

Under our assumption that the public good is a normal good, moreover, the level of voluntary provision of the public good will be greater the more extreme the inequality of wealth. The provision will be lowest when both contribute. If we drop the assumption that there are only two contributors, we get the paradoxical result that the greater the number of Nash-Cournot voluntary contributors, the lower the supply of the public good. The greater provision of public goods when there is greater inequality will also prove to be an important point in our later analysis of an ideology that was common in "feudal" times and is by no means absent today. This consideration also makes it obvious that the greater the inequality, the greater the resource cost of an inequality-reducing gift of the public good to the poorer

party.[6] Thus the natural egalitarianism of public good provision is in some sense stronger the greater the degree of inequality.

XII

The situation is usually the same when there are differences in preferences, but the natural egalitarianism is complicated or occasionally even offset by a neglected tendency for the exploitation of the zealous by the apathetic. Suppose that the parties have identical initial endowments of private wealth but differ in their preferences for the public good. Then the party with the strongest preference for the collective good will necessarily bear a disproportionate share -- and often all -- of the costs of providing it. The greater the difference in tastes for the public good, the larger the amount that will be provided. This logic helps explain the importance of fanaticism for collective action.

Nonetheless, so long as collective goods have a positive income elasticity of demand, the natural egalitarianism of voluntary public gods provision will tend to apply, and it will necessarily apply in any situations where the differences in wealth are great enough for the richest party to have the largest absolute demand for the public good. In any such situation, the richest party will necessarily bear a disproportionate share -- and often all -- of the cost of providing the collective good.

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[6] We cannot analyze the value of the public good gift in terms of the poorer party's willingness to pay, for he is not willing to pay anything for additional amounts of the public good. When his income is very low, even his willingness to pay for private goods is constrained by his poverty.

XIII

We have now been able to see why the Warr neutrality has rightly been described in the literature as "startling" and "striking," and also why no empirical illustrations of it have been recounted in the literature. We have also now been able to see the sweeping character of the natural egalitarianism in voluntary burden sharing with public goods. Natural egalitarianism applies not only to the situations where the party with the largest absolute demand for the collective good bears all of the costs, but even in the special cases where the Warr neutrality result holds.

The foregoing analysis also bears on a number of historic and contemporary debates about public policy. In prior centuries it was often argued, as it is on some occasions even today, that arrangements that generate huge fortunes are desirable, not simply on grounds of incentives, but also on the grounds that the super-rich will voluntarily provide public goods for the society, such as philanthropy, patronage of artists, and so on. Aarts and Lieshout (1986) have, for example, made this point. In some earlier times, the richest sometimes also financed the police power needed to maintain or order.[7] Quite apart from the obvious distributional objections to such arrangements, they are not efficient, since Nash-Cournot voluntary provision undoubtedly does not supply Pareto-optimal quantities of public goods, even when inequalities are so great that there is some voluntary provision. It is nonetheless

[7] Once the richest person buys the coercive power needed to maintain order, he also has an incentive to tax the rest of the population for his own benefit. Historically, this has been a common occurrence.

true, as this paper has shown, that the level of public goods provision does increase with the degree of inequality.

Another policy implication of the argument here has a more contemporary ring. The theoretical demonstrations in the literature cited earlier in this paper, to the effect that the governmental provision of public goods will offset private contributions dollar for dollar, are almost certainly not relevant to the world we live in -- they assume, as a starting point, the conditions needed for Warr's neutrality result. Both empirical observation and the theoretical argument offered in this paper indicate that this result, if it occurs at all, is applicable only in rare cases. Thus many of the conclusions about public expenditures and private charity that are offered in the neutrality literature normally have no pertinence for the real world. Most notably, there is no warrant whatever for the conclusion that the high levels of public expenditure on welfare and defense in the present century have not brought about any net increase in aid to low income people or in military expenditures. Neither is it reasonable to conclude, as Bernheim does, that the implausibility of the predictions of his model is a sign that individuals do not have normal economic motivation with respect to public goods; the predictions are, instead, implausible because they take the conditions for Warr's neutrality result, which rarely if ever hold true for any country, as a starting point. A proper understanding of the issues raised in the Warr literature does not in any way call into question the familiar idea that *laissez faire* will lead to the severe underprovision of public goods, and that government provision can increase the supply, even if there are at the same time private contributions to the provision of these public goods.

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